JOURNAL OF AIRCRAFT
Vol. 29, No. 3, May–June 1992

Engineering Notes

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Coalescence of Aeroelastic Modes in Flutter Analysis

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Introduction

FLUTTER due to coalescence of modes occurs in aeroelastic analysis when the aerodynamic loads are limited to those proportional to the displacements.[1-3] It has been demonstrated that in-quadrature airloads have little effect on the flutter condition when this is severe,[4] and their effect is only important for the milder form of flutter. Current aeroelastic analyses are performed by numerically tracing the problem eigenvalues vs the velocity. The system stability is then studied through mode coalescence if damping is omitted, or the point of zero damping, if this is considered. This corresponds to the solution of a complex eigenvalue problem for each iteration step.

This note treats flutter due to coalescence of modes. It is shown that the stability of the system can be studied in a concise form and the solution of the complex eigenvalue problem can be avoided. It is also shown that the determination of the critical condition is reduced to that of the solution of two simultaneous equations in two unknowns, namely the flutter velocity and the flutter frequency.

Problem Formulation

Flutter due to coalescence of modes occurs in aeroelastic analysis when the action of the air forces is limited to those in phase with the displacements. Under such conditions, the equation of motion can be written as

$$[\mu[I] + [S] + \Lambda[A]]\{q\} = \{0\} \tag{1}$$

where $[S]$ is the dynamic matrix, $[A]$ is the aerodynamic matrix, Λ is the aerodynamic pressure parameter, μ is the aeroelastic eigenvalue, and $\{q\}$ is the vector of the generalized coordinates. Equation (1) represents a parametric eigenvalue problem, with Λ considered as the parameter of the problem.

Not all the modes of Eq. (1) can be contribute to a fluttering condition. Flutter occurs physically due to the interaction between at least two degrees of freedom.[5] Mathematically, this is expressed through the coupling of terms of the matrix $[A]$ of Eq. (1). Therefore, the problem can be simplified by eliminating from Eq. (1) all the individual modes having null columns and/or rows except for a possible nonzero diagonal element. These modes do not contribute to or affect the fluttering condition. They can only give rise to static divergences, and these can easily be treated through the solution of their individual single-degree-of-freedom equations. Separate, then, the remaining modes into groups that interact aerodynamically. Each of these groups represents a system of equations similar to Eq. (1), but of reduced order, and can be treated separately for examination of the aeroelastic stability.

Examine under what condition each of these smaller systems are flutter prone. As the value of Λ increases, the roots of the characteristic Eq. (1) can be complex, and in such a case they will appear as conjugate pairs, because the coefficients of the characteristic equation are real. Under such conditions, one of each root pair will give an unstable motion. The borderline of stability will be obtained at the point of coalescence of two roots.

Now, the characteristic equation of each reduced system can be written as

$$\mu^n + c_1\mu^{n-1} + \cdots + c_{n-1}\mu + c_n = 0 \tag{2}$$

where n is the number of modes with aerodynamic interaction. The coefficients of the characteristic Eq. (2) read

$$\begin{aligned} c_1 &= T_1 \\ c_j &= -\frac{1}{j}[c_{j-1}T_1 + c_{j-2}T_2 + \cdots + c_1T_{j-1} + T_j] \\ j &= 2, 3, \ldots, n \end{aligned} \tag{3}$$

where T_j are the traces of the power j of the reduced $[S + \Lambda A]$ matrix. Let the roots of Eq. (2) be μ_i where $i = 1, 2, \ldots, n$; and using the Encke notation[6] the coefficients (3) are written as

$$c_j = [\mu_i^j] \tag{4}$$

On the borderline of stability, Eq. (2) has a pair of repeated roots. Let this pair of roots be $\bar{\mu}$ and denote the rest of the roots by ν_l, where $l = 1, 2, \ldots, n - 2$. Doing so, the coefficients of Eq. (4) read

$$c_j = \bar{\mu}^2[\nu_i^{j-2}] + 2\bar{\mu}[\nu_i^{j-1}] + [\nu_i^j] \tag{5}$$

where, again, the Encke notation has been used, with $[\nu_i^0]$ and $[\nu_i^j] = 0$ for $j < 0$ and $j > n - 2$. Equation (5) represents a system of n equations in n unknowns, namely, $\bar{\mu}$, Λ, and $[\nu_i^j]$. Eliminating the Encke roots $[\nu_i^j]$ from these equations, we obtain

$$c_{n-1} - \sum_{i=1}^{n-1}(-1)^{i+1}[i + 1]\bar{\mu}^i c_{n-i-1} = 0 \tag{6}$$

$$c_n - \sum_{i=1}^{n-1}(-1)^{i+1} i\bar{\mu}^{i+1} c_{n-i-1} = 0 \tag{7}$$

This represents two equations in two unknowns $\bar{\mu}$ and Λ. Their solution determines the fluttering parameters of the aeroelastic problem, namely, $\bar{\mu}_{fl}$ and Λ_{fl}.

Numerical Applications

As a first example, we consider the supersonic flutter of a simply supported square thin circular cylindrical panel with

Received May 7, 1990; revision received June 10, 1991; accepted for publication July 10, 1991.

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flow parallel to the straight edge. The Galerkin formulation is used with the free vibration modes taken as trial functions.

The dynamic matrix $[S]$ in Eq. (1) is diagonal and reads

$$[S] = \left[(m^2 + n^2\beta^2)^2 + \frac{\alpha m^4}{(m^2 + n^2\beta^2)^2} \right] \tag{8}$$

with α being the curvature effect given by

$$\alpha = \frac{12(1 - \nu^2)}{\pi^4} \left(\frac{a}{h}\right)^2 \left(\frac{a}{R}\right)^2$$

Other notations are as given in Ref. 1.

Notice that the equations of motion are uncoupled aerodynamically in the cross-stream direction due to the orthogonality properties of the trigonometric functions, i.e., no coalescence of modes with different n can occur. Furthermore, due to the natural frequency spectrum of the curved panels, the lowest modes are not necessarily for $n = 1$. Moreover, due to the aerodynamic coupling and the frequency spectrum, the lowest coalescence for a given n is not necessarily for $m = 1$ and 2. Therefore, the solution for a given n value will be done by computing the critical values for minimum coalescence. Convergence of the solution is then studied by increasing the number of trial functions used in the Galerkin formulation. The numerical computation is performed for a value of $\alpha = 10^3$. The results obtained are summarized in Fig. 1. From this Fig. it can be observed that the minimum critical parameters, for the case here considered, are for $n = 1$ and for the second coalescence, i.e., $m = 3$ and 4. At a higher dynamic pressure, as shown in Fig. 1, we have coalescence of the first modes, i.e., for $m = 1$ and 2 and for $n = 4$.

As a second example, we consider the hypersonic flutter of the cantilever wing studied in Ref. 7 and reported in Ref. 1. The aeroelastic analysis of Ref. 1 considered the first three

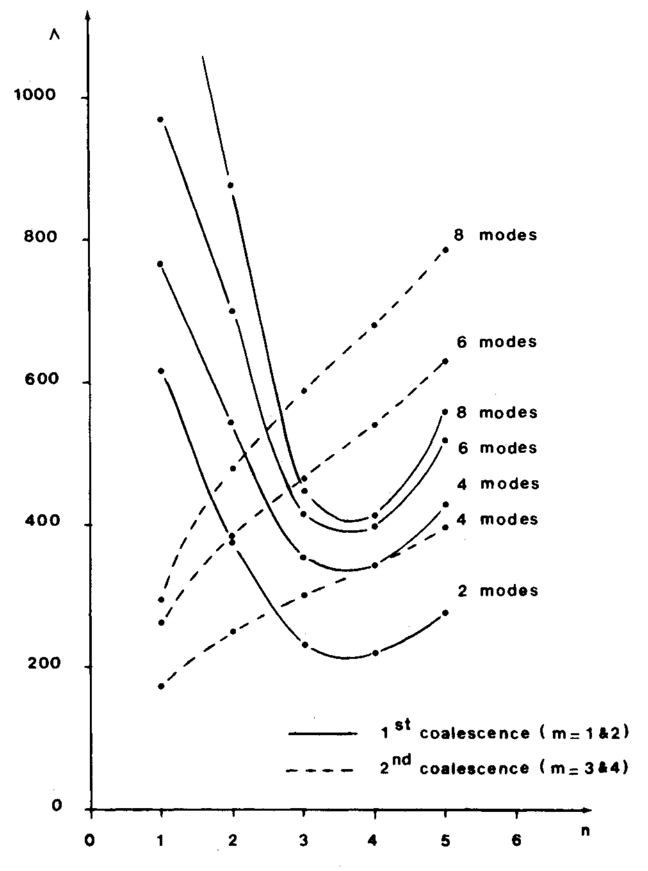

Fig. 1 Flutter boundaries for a square thin cylindrical panel.

natural modes, namely, first bending, first torsion, and first chordwise modes, based on experimental measurements and piston theory for the aerodynamics. Using the data given in Ref. 1 and omitting the aerodynamic damping, the aeroelastic equations of motion are again given by Eq. (1), where now

$$\mu = -\left(\frac{\omega}{\omega_2}\right)^2 \qquad \Lambda = \left(\frac{U}{b\omega_2}\right)^2 \qquad [S] = [\omega_i^2/\omega_2^2]$$

$$[A] = \begin{bmatrix} 0 & -0.0425 & -0.00668625 \\ 0 & -0.0075 & -0.05517185 \\ 0 & 0 & -0.01125 \end{bmatrix} \tag{9}$$

Other notations are as given in Ref. 1. This analysis was done for several values of the chordwise frequency ratio, $\Lambda_3 = \omega_3/\omega_2$. By examining the matrix $[A]$ given in Eq. (9), it can be concluded that based on the frequency coalescence theory, no flutter can take place, since there is no aerodynamic coupling. Only static divergence can occur, and this is easily obtained by solving

$$-\mu + 0.2195 = 0 \tag{10}$$

$$-\mu + 1 - 0.0075\,\Lambda = 0 \tag{11}$$

$$-\mu + \Lambda_3^2 - 0.01125\,\Lambda = 0 \tag{12}$$

for $\mu = 0$. This shows that the bending mode has no contribution to static divergence, and the torsion and the chordwise modes act independently with divergence values of Λ given by Eqs. (11) and (12) which are the values reported in Ref. 1. Furthermore, it is interesting to observe that for certain values of Λ and Λ_3, Eqs. (11) and (12) can lead to equal values of μ. However, this is not a flutter condition; there is no aerodynamic coupling and therefore no frequency coalescence. This is only frequency crossing. Now, if damping is included, but the damping due to curvature is omitted, the following imaginary matrix is added to the equation of motion [Eq. (1)]:

$$[A_1] = 0.03125\, i(\Lambda\mu)^{1/2}[I]$$

where $[I]$ is the identity matrix. Again, no flutter condition can take place, since there is no aerodynamic coupling between the modes. Now, if the damping due to curvature is considered, the following imaginary matrix is futher added to the equation of motion [Eq. (1)]:

$$[A_2] = i(\Lambda\mu)^{1/2} \begin{bmatrix} 0 & 0.00339 & 0 \\ 0.00509 & 0 & 0.001974 \\ 0 & 0.0065 & 0 \end{bmatrix}$$

Now, consider the aerodynamic coupling between modes 1 and 2 and modes 2 and 3, which are the types of flutter reported in Ref. 1 and can be explained through the damping due to curvature. The first mode of flutter (bending-torsion), as shown in Ref. 1, is a mild type of flutter since the damping curve crosses the zero line twice or even decreases and then increases again without crossing the zero line. The second mode of flutter is a violent one and is due to coupling between torsion and chordwise modes. This type of flutter cannot be explained as being caused by the damping effect resulting from curvature. If the mode shapes used in Ref. 1 are examined, it can be shown that the chordwise mode has been approximated by a mode with two nodal lines that run parallel to the leading edge. This is done to preserve orthogonality and to approximate the measured values. It is known that the first chordwise mode of a cantilever homogeneous square plate has nodal lines that approach the leading and trailing edges at the root and run away from these edges moving spanwise. Furthermore, exact symmetry of nodal lines in relation to midchord is preserved only if the plate is theoretically ho-

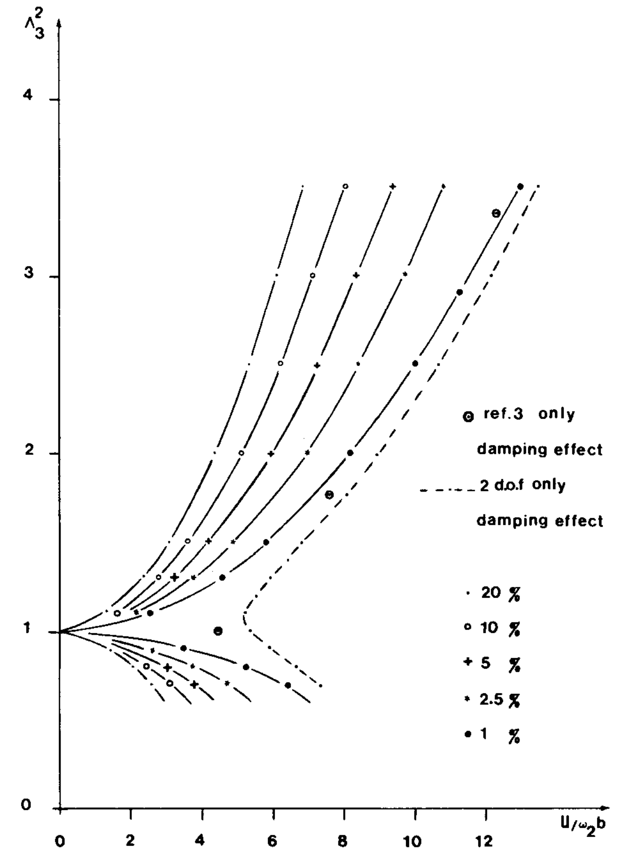

Fig. 2 Flutter dynamic pressure vs chordwise natural frequency of a rectangular wing.

mogeneous. The measured value of the frequencies of Ref. 7 reveals that the plate was not homogeneous, its chordwise frequency being much less than that theoretically predicted for a square plate, i.e., it approaches the value for a plate with an aspect ratio 1:2.

In order to examine the tendency of this mode to flutter when coupling with the torsion mode, a small disturbance in the aerodynamic stiffness matrix is included. This disturbance is accomplished by introducing in the aerodynamic stiffness matrix a value for the element (3,2) which is equal to a negative percentage of the element (2,3). The result of the analysis is shown in Fig. 2 for various percentages. Notice that a 10% disturbance is equivalent to moving the nodal lines only 1% chordwise forward and an asymmetry of 5% in the deflection of the trailing edge in relation to the leading edge. Observe that only 1% disturbance without the aerodynamic damping shows evidence of this type of violent flutter. Figure 2 also shows the flutter parameters given in Ref. 1 for an analysis when only modes 2 and 3 are considered. Finally, it must be observed that values of $\Lambda_3 \leq 1$ are physically impossible, since the chordwise mode cannot be a graver mode than the torsion mode. This classical example has been analyzed in detail in order to show the importance of the accuracy in the mode shapes when used in flutter analysis and the adequacy of the mode coalescence theory to predict violent types of flutter.

Conclusions

A method has been presented for the solution of the flutter problem within the theory of mode coalescence. It has been shown that the solution of the complex eigenvalue problem can be avoided and the problem can be reduced to that of the solution of two simultaneous algebraic equations with the critical flutter parameters as unknowns. Numerical applications have been presented and the adequacy of the use of the mode coalescence theory has been discussed.

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Structural and Aerodynamic Data Transformation Using Inverse Isoparametric Mapping

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I. Introduction

FINITE element methods are being used in aeroelastic analysis to predict flutter boundaries, response, etc. In aeroelastic analysis, different discretization procedures are used for structural and aerodynamic parameters. For example, in MSC/NASTRAN, one set of discretization is based on aerodynamic theory and the other set of discretization is based on structural considerations. It is necessary to find the aerodynamic parameters at the structural grid and vice versa for aeroelastic analysis. For example, the structural deformations are required at the aerodynamic grid points and the aerodynamic loads are required at the structural grid points.

Several numerical procedures were developed to get the necessary information between the aerodynamic and the structural grids. These procedures employ a least squares technique for interpolation,[1] polynomial fit,[2] and spline functions based on simple beam and infinite plate equations.[3,4] In Ref. 5, a piecewise cubic monotone interpolation scheme was used to determine the displacements and the slopes at the aerodynamic grid points by deriving a transformation matrix. However, in all of these procedures, a system of equations has to be solved to get the required information between the aerodynamic and structural grid points. For irregularly shaped structural or aerodynamic grids, the system of equations becomes ill conditioned and the results will be inaccurate.

The present approach is to use the inverse isoparametric mapping to transform the state variables such as displacement, load, stress, pressure, temperature, etc., from the structural grid points to aerodynamic grid points. The plane form of the wing can be represented by either four-node or eight-node isoparametric finite elements. Although the present approach is intended for interpolation between the two sets of grids, extrapolation to control surfaces, such as flaps, is possible by combining the well-known extrapolation techniques and inverse mapping procedure. The present approach is demon-

Received Jan. 27, 1990; revision received Oct. 15, 1990; accepted for publication Dec. 31, 1990.

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